Calibration of a Weather Radar by Using a Standard Target

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ABSTRACT

A simple method is described for calibrating a weather radar by means of a standard spherical target, thus permitting the radar to be used for quantitative measurements of storm reflectivity. The technique involves determination of that storm reflectivity which provides an echo equivalent to that from the known target. The sphere, suspended from a balloon, is tracked as it leaves the radar site. Its echo is "measured" by reducing the receiver gain control to the threshold of visibility. The threshold gain setting is thereby calibrated and subsequently provides an accurate measure of storm reflectivity. There is no need for any other test equipment such as a microwave-signal generator. Absolute accuracy is greater than that attainable with a signal generator since no reliance need be placed on the generator calibration or upon the specified antenna gain.

1. Introduction

The importance of quantitative radar measurements of a storm's intensity need hardly be stated. Some research projects, realizing this, have gone to great pains to calibrate their radars (Austin and Williams, 1951; Donaldson, 1958). Operationally, however, we are still in the dark ages. Storm intensities are still reported as "weak, moderate or strong" as judged subjectively from the echo brilliance on the PPI scope. Such qualitative assessments are inaccurate, lead to conflicting reports from radars of varying sensitivity, and do not allow a reliable assessment to be made of storm intensity. A principal reason for the continued use of such an antiquated system has been the lack of proper test equipment with which to calibrate the radars. This paper describes a simple calibration technique which avoids the use of an expensive signal generator. The scheme should be of use to all those who employ radar either operationally or in research.

The method involves the use of an expendable balloon-borne aluminium sphere, the relation between its echo and that to be expected from precipitation of known reflectivity, and a graduated dial added to the receiver gain-control knob. No signal generator is required unless one wishes to express the echo intensities in absolute power units. Indeed, since the absolute accuracy of most signal generators is of the order of ±3 dB, the standard spherical target is necessary to obtain improved accuracy. Also, the artificial echo from the signal generator is usually inserted into the radar via a directional coupler in the waveguide, thus requiring reliance upon the theoretical or rated antenna gain. This factor, too, is accounted for in the spherical target method.

2. Theory

The radar-range equation for any point target is well known (Kerr, 1951; p. 35)—

\[
P_r/P_t = \left(\frac{G_o\lambda^2 F^4}{(4\pi)^2}\right)\sigma/r^4
\]

(1)

where the factors have the following meaning and units: \(P_r = \) echo power (watts), \(P_t = \) peak transmitted power (watts), \(G_o = \) antenna gain over an isotropic radiator along the beam axis, \(\sigma = \) target back-scatter cross-section (cm\(^2\)), and \(r = \) target range (cm). The factor \(F\) is a general term which takes into consideration the effect of the target being off the beam axis and losses due to atmospheric attenuation and refraction. Atmospheric attenuation is negligible at wavelengths of 3 cm and longer, provided the tests are conducted in the absence of precipitation. Also, since the present method involves peaking of the signal by centering the beam on the target and is accomplished at relatively high elevation angles where refraction is negligible, \(F\) may be taken as unity.

In the case of hydrometeors distributed through a storm so as to fill the radar beam,
their equivalent cross-section is
\[ \sigma = \eta V. \] (2)

Here \( \eta \) is the reflectivity of the particles in \( \text{cm}^{-1} \), and \( V \) is the pulse volume given by
\[ V = \pi r^2 \theta \phi h / 8 \] (3)

where \( \theta \) = horizontal beam width (radians), \( \phi \) = vertical beam width (radians), and \( h \) = the pulse length in space (cm).

However, with precipitation which is distributed throughout the beam, we note that those hydrometeors which are off the beam axis are illuminated with somewhat less intensity than those along the axis. This may be taken into account either by using the average antenna gain \( G \) instead of the axis gain \( G_0 \) or by following the method of Kerr which is to employ the average value of \( F \). Austin and Williams (1951) have shown that \( F^4 \) averaged across the beam of the SCR-615 radar is 0.83. This value should be about the same for any symmetrical pencil beam. Thus, precipitation must have a cross-section greater by a factor of 1.2 than that of a point target in order to return the same echo power.

Another factor to be considered in determining the equivalence between the echoes from precipitation and a point target is the difference in their statistical behavior. The echoes from a single spherical reflector are perfectly steady or coherent, while those from precipitation fluctuate rapidly with the amplitudes of a large enough sample of pulses fitting a known probability distribution (see Marshall and Hitschfeld, 1953). In the present method, the echoes are compared at their threshold of visibility on either the PPI or RHI scope. The net effect of integration by both the scope and the eye will be to respond to the average amplitude of the echoes. For the incoherent precipitation, this will be 0.89 \( P^4 \) (Austin and Williams, 1951), where \( P \), the average echo intensity, is directly proportional to storm reflectivity. Of course, the steady echoes from the sphere will give a threshold signal of exactly \( P^4 \). Therefore, in order to provide the same effective threshold amplitude, the average echo intensity from the storm must be greater than that from the sphere by \( (0.89)^{-2} = 1.27 \).

Together, the effect of the tapered beam and the incoherent nature of the precipitation echoes requires that the precipitation have a cross-section of \( 1.2 \times 1.27 = 1.52 \) times that of a point target in order to provide equal threshold amplitudes. This equivalence may be expressed as
\[ \sigma_p = 1.52 \sigma_s \] (4)

where the subscripts signify "precipitation" and "sphere" respectively. By using (2), the equivalent precipitation reflectivity is
\[ \eta = 1.52 \sigma_s / V. \] (5)

If one wishes to express the storm intensity in terms of the more usual reflectivity factor \( Z = \Sigma n_i D_i^6 \), the summation of the sixth powers of the drop diameters in a unit volume, use may be made of the well known relationship for Rayleigh scattering
\[ \eta = 10^{-12} (\pi^5 / \lambda^4) K^2 Z \] (6)

where \( K = (m^2 - 1) / (m^2 + 2) \) and \( m \) is refractive index of the scatterers. The factor \( 10^{-12} \) is used here so that \( Z \) and \( \eta \) may be expressed in their conventional units of \( \text{mm}^2 / \text{m}^3 \) and \( \text{cm}^{-1} \) respectively. It must be emphasized that eq (6) assumes that the echoing particles are small with respect to the radar wavelength. When large particles such as hailstones are present, the use of (6) provides only an "apparent" or equivalent \( Z \). Values of \( K^2 \) are given by Gunn and East (1954) for ice (0.197) and for water (0.93, for wavelengths of 3 to 10 cm and temperatures of 0C to 20C). For later use, we therefore have for water particles
\[ \eta = 5.96 \times 10^{-13} Z \quad \text{at} \quad 4.67 \text{ cm}, \]
\[ \eta = 2.84 \times 10^{-14} Z \quad \text{at} \quad 10 \text{ cm}. \]

### 3. Practice

Since the present method assumes that a signal generator is not available, it is apparent that we must compare the precipitation echo with that from the standard target. Indeed, this is desirable in any case, since, as has been mentioned, the absolute accuracy of most signal generators cannot be relied upon. There are various techniques for making such a comparison. One of the most reliable methods is to reduce the receiver gain control (usually in the intermediate frequency or IF stages) until the echo just barely illuminates the PPI or RHI scope. This threshold gain setting, as read from the dial or, more reliably, the IF gain bias voltage, is a reproducible measure of the echo intensity provided that the video gain and scope brilliance controls are held constant. In other words, when one measures a precipitation echo, he is simply comparing the threshold gain with a corresponding gain setting previously measured for the standard target.
**Table 1. Characteristics of the MPS-4 and Type 13 radars.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>MPS-4</th>
<th>Type 13</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_t$ = peak transmitted power</td>
<td>watts</td>
<td>$1.4 \times 10^5$</td>
<td>$5 \times 10^5\star$</td>
</tr>
<tr>
<td>$P_{\text{min}}$ = minimum detectable echo power</td>
<td>watts</td>
<td>$10^{-14}$†</td>
<td>—</td>
</tr>
<tr>
<td>$G_a$ = peak antenna gain</td>
<td>deg</td>
<td>4</td>
<td>7.5</td>
</tr>
<tr>
<td>$\theta$ = horizontal beam width</td>
<td>deg</td>
<td>0.8</td>
<td>1.5</td>
</tr>
<tr>
<td>$\phi$ = vertical beam width</td>
<td>meters</td>
<td>390</td>
<td>600</td>
</tr>
<tr>
<td>$h$ = pulse width</td>
<td>cm</td>
<td>4.67</td>
<td>10.0</td>
</tr>
<tr>
<td>$\lambda$ = wavelength</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

**Theoretical estimate based on antenna dimensions and beam widths.**

* Nominal value. † Measured.

The scheme will be illustrated for two radars which were calibrated simultaneously using the same standard target. The radars were the 4.67-cm MPS-4 and the 10-cm A.M.E.S. Mk I (type 13), both operated with RHI scans and located at the East Hill Radar Station of the British Meteorological Service, near Dunstable, Bedfordshire. The radar characteristics are given in table 1. The MPS-4 was equipped with a voltmeter to read the IF gain bias, while only an arbitrarily graduated dial was available on the Type 13 gain control.

A 24-inch diam spun-aluminium sphere, whose scattering section is simply the geometrical cross section of $2.92 \times 10^3$ cm$^2$ and whose weight was about 2.4 kg, was attached to a 500-g rubber balloon. The balloon was inflated with hydrogen so as to have a free lift of about 150 g and a rising speed of about 250 ft per min, sufficient to ensure that, after release from the radar site in the existing winds of 10–15 kn, the target would not rise above the maximum elevation (20 deg) of the radars but yet would rise quickly enough for its echo to become free of ground clutter at short ranges, for as we shall see it is desirable to make measurements at the smallest possible ranges. The radars followed the balloon and scanned in their normal RHI mode. An occasion was chosen when the wind direction in the lowest 15,000 ft was nearly uniform with height, since then the azimuth of the balloon remains nearly constant, a great convenience when the radar beams must continually be centered upon the target. For this reason, it is also desirable to scan in a mode perpendicular to the direction of the largest beam width—i.e., PPI with a wide vertical beam and RHI with a wide horizontal beam. In order to ensure further against the loss of the target, the balloon was tracked visually by theodolite, and periodic readings of azimuth and elevation were given to the radar operators by telephone. Measurements were begun as soon as the radar operators could clearly distinguish the target echo from the ground clutter. With both radars, the operators would attempt to maximize the target echo by adjustment of the azimuth, perhaps with the gain somewhat reduced, and would then further reduce the gain until the echo was barely detectable. In the MPS-4 radar, a second observer recorded the IF threshold gain bias along with time, azimuth, and target range. This had the small advantage of preventing the operator on the gain control from biasing his readings according to previous ones. Readings were taken as frequently as possible by using small azimuth adjustments in order to ensure that at least a few of the measurements were made along the horizontal beam axis. By scanning in the vertical across the target, we were always certain to obtain the maximum in that direction.

### 4. Calibration of the MPS-4 radar

In the case of the MPS-4 radar, we had previously determined by calibration with a microwave-signal generator that the IF gain bias was linearly related to the threshold signal strength in decibels, as indicated on the left- and right-hand ordinate scales of fig. 1. Such linearity facilitates matters greatly and permits one to plot the threshold bias voltage *versus* range on a linear-log graph as shown in fig. 1. A line of best fit is then drawn close to the maximum strength signals, since smaller signals may be assumed to indicate that the target was not properly centered along the beam axis. A few signal intensities should be permitted to fall slightly above the line, since it is experimentally possible for the operator to misjudge the threshold point. The standard deviation of such variations is not likely to exceed ±0.5 db (Atlas, 1947). Therefore, there should not be more than a fraction of a decibel leeway in positioning the line of best
fit. When the slope of the threshold signal versus gain voltage is known, as it was for the MPS-4, the slope of the fitted line should be adjusted slightly to a perfect inverse fourth-power law between echo intensity and range. Such adjustment of the slope will be minor if a sufficient number of points is plotted. The resulting curve establishes the relation between threshold gain bias (signal strength) and range for the standard reflector. The curve must not be extrapolated beyond the range of observed gain settings, unless one is certain of the linearity of the gain versus signal-strength (db) curve.

The reader may be interested in a comparison of the expected theoretical return from the spherical target to that actually measured. The theoretical return is computed readily from eq (1), using the parameters of table 1. For the 24-inch sphere, the theoretical relation is \( P_r = 5.35 \times 10^{-7}/r^4 \) where \( r \) is in nautical miles. This falls just 6 db above the actual line, suggesting that our theoretical estimate of the antenna gain was high by a factor of 2.

By using eq (3), (4), (5), and (6), one then establishes the relation between \( Z \) and \( \sigma_p \) for his particular radar. For the MPS-4,

\[
Z = 5.09 \sigma_p/r^2
\]

where \( r \) is in nautical miles to agree with the radar-range calibration. It is to be noted that the accuracy of this relationship depends essentially on a knowledge of the size of the pulse volume, which is determined by the beam widths and pulse length eq (3). Since the latter parameters are usually known to be better than 10 per cent accuracy, the maximum possible error in (7) is about 30 per cent. Ordinarily, we would expect accuracy of the order of ±10 per cent here. By inserting \( \sigma_p = 2.92 \times 10^3 \) in (7), we then compute values of \( Z \) as a function of \( r \) along the calibration curve or \( \sigma_p \) locus. Isoptihs of constant \( Z \) are then drawn through these points using the relation \( P_r = \text{constant}/r^2 \); i.e., a 20-db reduction in echo intensity corresponds to a ten-fold increase in range. In other words, the slope of the \( Z \) isopleths is half that of the \( \sigma_p \) locus. Additional \( Z \) isopleths may be added, since \( P_r \) is directly proportional to \( Z \).

The calibration chart is now completed and ready for use on rainstorm measurements. However, it is to be noted that the use of the chart is restricted to the range of threshold gain settings employed in the sphere test unless the relationship between threshold gain and signal strength has been determined by calibration with a signal generator. Without the latter calibration, fig. 1 indicates that it would not have been possible to make measurements on storms with \( Z \)'s in excess of \( 10^6 \) mm\(^6\) per m\(^3\). This would be a serious limitation, since such storms are the more dangerous ones. It is, therefore, important to obtain measurements on the spherical target over the maximum possible range interval. If we had been able to begin measurements at a range of 1 mi, the threshold bias calibration would have been extended to a level of about 37 v, or approximately 23 db higher in echo power. Alternatively, one may use a larger target in order to extend the calibration. Loci for 12-inch and 48-inch diam spheres have been added to fig. 1 to illustrate this effect. It is apparent that it is desirable to use the largest practicable target and to commence measurements at the shortest possible ranges. In order to obtain such short ranges, it may be more convenient to release the balloon a few miles upwind of the radar. Obviously, since the range over which the receiver gain may be varied is limited, it will ordinarily be impossible to measure some of the more intense storms at short ranges. A means of extending the range of utility of the calibrated portion of the gain-control dial is described in the next section.
Of course, if there is a non-linear relationship between gain setting and threshold echo intensity, the point will describe a curve on a linear-log graph such as that in fig. 1. The $Z$ loci would also be curved. This makes construction of the chart rather more difficult. To illustrate a simple approach for such non-linear receivers, we describe the calibration of the 10-cm Type 13 radar.

5. Calibration of the Type 13 radar

This radar was equipped only with a gain-control knob having a pointer and an arbitrary scale. The threshold gain settings were plotted as a function of range as in fig. 2. A smooth curve was then drawn through the minimum gain (maximum signal) values permitting just a few points to fall on the other side, to allow for observer error. For this radar, we find the precipitation equivalent of a point target is

$$Z = 25.6\sigma_p/r^2$$  \hspace{1cm} (8)

where $r$ is in statute miles. After substituting $\sigma_p = 2.92 \times 10^4$ cm$^3$ for the 24-inch diam sphere, we plot the resulting locus $Z = 7.48 \times 10^4/r^2$ on a log-log graph as in fig. 3. Along this locus, we know the threshold gain settings as a function of range according to the smooth curve of fig. 2. Tick marks may thus be placed along the $\sigma_p$ locus with numerals indicating the corresponding gain values. Since each gain setting represents a specific (but unknown) echo power, we may then draw loci of constant gain using the relation $Z = \text{constant} \times r^2$. It is evident that this method overcomes any problem with non-linearity of the receiver response. Such non-linearities simply appear in the irregular spacing between adjacent gain lines.

Once again, the $\sigma_p$ loci for 12 inch and 48 inch diam spheres have been added to indicate the slight advantage of a larger sphere. As before, it is seen that it is desirable to get measurements at the shortest possible range in order to extend the calibration as far as possible. While measurements were not actually begun until the target reached a range of about 3 mi, the data of fig. 2 permit us to extrapolate slightly into a range of 1.8 mi and a corresponding gain setting of 16. We could not have extended the calibration much further even if we had begun readings at a range of 1 mi.

It is apparent that intense storm $Z$'s of about $10^8$ mm$^3$ per m$^3$ or higher could not be measured in the calibrated range of gain settings. To overcome this difficulty, one could readily insert a fixed IF attenuator in the receiver when the gain setting had to be reduced below a level of 16. The amount of such attenuation is readily computed as follows. Gain 16 occurs at a range of 1.8 mi, while gain 26 occurs at 42 mi. This represents a power ratio of $40 \log (42/1.8) = 54.7$ db. Insertion of such an attenuator would bring the 26 gain locus up coincident with the 16 locus, and the chart would simply be repeated above. Of course, any known attenuator of smaller value could be inserted and the calibration extended accordingly.

6. Receiver recovery time correction

There is one additional factor to consider in the application of the above methods. This is the variation of threshold signal intensity with range due to the recovery time of the receiver.
If the radar recovery time extends significantly beyond the minimum range at which the sphere was observed, it may be considered as follows. The recovery time curve is plotted as a function of range as in the inset diagram in fig. 3. In the case of the Type 13 radar, we have assumed a reasonable recovery time according to our experience; i.e., it was known that the receiver was essentially fully recovered at 2 mi and was about 3 db down at 0.6 mi. In order to correct the gain 16 line, for example, one notes that the storm reflectivity which will provide the identical threshold gain to that of the sphere according to eq (8) is only accurate at the range of the sphere, or 1.8 mi. Since the receiver is about 0.9 db less sensitive at 1 mi, the equivalent Z will have to be 1 db greater than expected from (8) in order to give the same threshold. At ranges beyond about 3 mi, the receiver has recovered to full sensitivity, about 0.3 db greater than that at 1.8 mi. Thus, beyond 3 mi range, storm Z's would have to be 0.3 db smaller than expected from eq (8) in order to give the same threshold gain. A smooth curve may be drawn through these points, intersecting the straight \( Z - r^2 \) line at 1.8 mi. In the case of the Type 13 radar, the true gain 16 (dashed) line is hardly distinguishable beyond the intersection point from the uncorrected one. There will be even smaller differences in the other gain isopleths (except at extremely short ranges), since the sphere was observed at distances at which the receiver was essentially fully recovered.

The receiver recovery time may be determined either with a signal generator or an echo box in the manner described in the radar handbook. If neither of these instruments is available, the following scheme is recommended. In a situation of widespread generally uniform precipitation, adjust the receiver so that the precipitation echo at a range of, for example, 10 mi just reaches saturation on the A-scope. At a range of \( 2 \times 10 = 14.1 \) mi, the echo from the same precipitation will be just 3 db smaller, provided that the rainfall is light and attenuation is negligible. The corresponding echo amplitude on the A-scope thus establishes the scope calibration. The observations should be repeated a number of times at various ranges \( r \) and \( 1.41r \) and various azimuths until the observer is confident of the calibration. Having once established the A-scope calibration, similar measurements may be made on uniform precipitation at close ranges. The recovery time is given as the range at which the echo amplitude falls to 3 db below the inverse-square curve of amplitude versus range. If the echo from uniform precipitation follows the inverse-square-range law at ranges corresponding to the closest at which the sphere was observed, it is apparent that no recovery-time corrections are necessary.

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REFERENCES


